Friday, September 11, 2015

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Problem 5

Problem. Evaluate the expression $\arccos \frac{1}{2}$ without using a calculator.

Solution. The angle whose cosine is $\frac{1}{2}$ is $\frac{\pi}{3}$.

Problem 7

Problem. Evaluate the expression $\arctan \dfrac{\sqrt{3}}{3}$ without using a calculator.

Solution. The angle whose tangent is $\frac{\sqrt{3}}{3}$ is $\frac{\pi}{6}$.

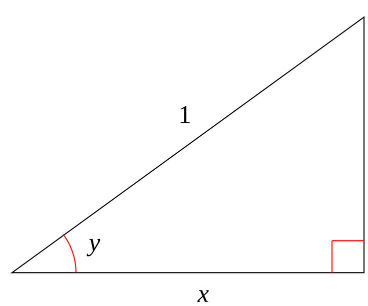
Problem 9

Problem. Evaluate the expression $\operatorname{arccsc}(-\sqrt{2})$ without using a calculator.

Solution. Because of the minus sign, the value must be in the fourth quadrant, where csc is negative. The value is $-\frac{\pi}{4}$.

Problem 15

Problem. Use the figure to write the expression $\cos y$ in algebraic form, given $y = \arccos x$, where $0 < y < \pi/2$.



Solution. $\cos y = x$.

Problem 17

Problem. Use the figure to write the expression $\tan y$ in algebraic form, given $y = \arccos x$, where $0 < y < \pi/2$.

Solution. The opposite side is $\sqrt{1-x^2}$, so $\tan y = \dfrac{\sqrt{1-x^2}}{x}$.

Problem 19

Problem. Use the figure to write the expression $\sec y$ in algebraic form, given $y = \arccos x$, where $0 < y < \pi/2$.

Solution. $\sec y = \dfrac{1}{x}$.

Problem 21

Problem. Evaluate each expression without using a calculator

(a) $\sin\left(\arctan\dfrac{3}{4}\right)$

(b) $\sec\left(\arcsin\dfrac{4}{5}\right)$

Solution. (a) Let $\theta = \arctan\frac{3}{4}$. The opposite side is 3 and the adjacent side is 4, so the hypotenuse is 5. Thus, $\sin\theta = \frac{3}{5}$.

(b) Let $\theta = \arcsin\frac{4}{5}$. The opposite side is 4 and the hypotenuse is 5, so the adjacent side is 3. Thus, $\sec\theta = \frac{5}{3}$.

Problem 22

Problem. Evaluate each expression without using a calculator

(a) $\tan\left(\arccos\dfrac{\sqrt{2}}{2}\right)$

(b) $\cos\left(\arcsin\dfrac{5}{13}\right)$

Solution. (a) Let $\theta = \arccos\frac{\sqrt{2}}{2}$. The adjacent side is $\sqrt{2}$ and the hypotenuse is 2, so the opposite side is $\sqrt{2}$. Thus, $\tan\theta = \frac{\sqrt{2}}{\sqrt{2}} = 1$.

(b) Let $\theta = \arcsin \frac{5}{13}$. The opposite side is 5 and the hypotenuse is 13, so the adjacent side is 12. Thus, $\cos\theta = \frac{12}{13}$.

Problem 23

Problem. Evaluate each expression without using a calculator

(a) $\cot\left[\arcsin\left(-\frac{1}{2}\right)\right]$

(b) $\csc\left[\arctan\left(-\frac{5}{12}\right)\right]$

Solution. (a) Let $\theta = \arcsin\left(-\frac{1}{2}\right)$. The opposite side is -1 and the hypotenuse is 2, so the adjacent side is $\sqrt{3}$. Thus, $\cot\theta = \frac{\sqrt{3}}{-1} = -\sqrt{3}$.

(b) Let $\theta = \arctan\left(-\frac{5}{12}\right)$. The opposite side is -5 and the adjacent side is 12, so the hypotenuse is 13. Thus, $\csc\theta = -\frac{5}{13}$.

Problem 24

Problem. Evaluate each expression without using a calculator

(a) $\sec\left[\arctan\left(-\frac{3}{5}\right)\right]$

(b) $\tan\left[\arcsin\left(-\frac{5}{6}\right)\right]$

Solution. (a) Let $\theta = \arctan\left(-\frac{3}{5}\right)$. The opposite side is -3 and the adjacent side is 5, so the hypotenuse is $\sqrt{34}$. Thus, $\sec\theta = \frac{\sqrt{34}}{5}$.

(b) Let $\theta = \arcsin\left(-\frac{5}{6}\right)$. The opposite side is -5 and the hypotenuse is 6, so the adjacent side is $\sqrt{11}$. Thus, $\tan\theta = -\frac{5}{\sqrt{11}}$.

Problem 25

Problem. Write the expression $\cos(\arcsin 2x)$ in algebraic form.

Solution. Let $\theta = \arcsin 2x$. Then the opposite side is $2x$ and the hypotenuse is 1. So the adjacent side would be $\sqrt{1-4x^2}$ and $\cos\theta = \sqrt{1-4x^2}$.

Problem 26

Problem. Write the expression $\sec(\arctan 4x)$ in algebraic form.

Solution. Let $\theta = \arctan 4x$. Then the opposite side is $4x$ and the adjacent side is 1. So the hypotenuse would be $\sqrt{1+16x^2}$ and $\sec\theta = \sqrt{1+16x^2}$.